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# RECENT DEVELOPMENTS IN COUPLING TOPOGRAPHICAL AND METEOROLOGICAL EFFECTS WITH THE GREEN'S FUNCTION PARABOLIC EQUATION (GFPE): THEORY AND EXPERIMENTS

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## **Abstract**

The paper deals with long range outdoor sound propagation predictions under range-dependant complex environment using the parabolic equation approach (PE). New developments allowing to work beyond the classical limits of the PE method are presented: the reflections due to vertical obstacles using a complementary Kirchhoff approach, the integration of complex noise barriers effects by the help of a hybrid BEM-GFPE model (Boundary Element Method – Green's Function Parabolic Equation), the reference rotation principle applied to embankments. These improvements have been implemented in a single code named ATMOS (Advanced Theoretical Models for Outdoor Sound propagation). Numerical examples of road traffic configurations illustrating such realistic combined effects are presented and compared to reference calculations as well as scale model measurements.

## **NUMERICAL MODEL**

Long range sound propagation with meteorological effects is investigated here by use of the Green's Function Parabolic Equation (GFPE) adapted by Gilbert [1] for outdoor acoustics. Starting from the Helmholtz equation in cylindrical coordinates (r,z) for sound pressure  $P(r,z) = \frac{1}{\sqrt{r}} u(r,z) e^{jk_r r}$ :

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + k(r,z)^2\right)P(r,z) = 0,$$
(1)

an initial field is propagated step by step from the source to the receiver. After developments described by Gilbert [2] the field at  $u(r + \Delta r, z)$  can be written as:

$$u(r + \Delta r, z) = \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} (U(r, k') + R(k')U(r - k')) \times e^{j\Delta r \left(\sqrt{k_r^2 - k'^2} - k_r\right)} e^{jk'z} dk' + 2j\beta \times U(r, \beta) \times e^{j\Delta r \left(\sqrt{k_r^2 - \beta^2} - k_r\right)} e^{-j\beta z} \right] \times e^{j\frac{\Delta r \delta k^2(z)}{2k_r}}$$

$$(2)$$

where  $U(r,k) = \int_{0}^{+\infty} e^{-jkz'} u(r,z') dz'$ ,  $\beta = \frac{k_r}{Z_g}$  and  $Z_g$  is the normalized ground impedance.

The GFPE is a powerful computational method able to calculate sound propagation in inhomogeneous atmospheres at long ranges. Even if this method permits to deal with varying ground impedance and simple obstacles (steps, rigid straight barriers), it shows a number of limitations intrinsic to the PE itself concerning: single and multiple reflections due to vertical obstacles, effect of complex noise barriers or complex geometries in the source or receiver vicinity, and configurations with uneven terrain and embankments.

#### SINGLE AND MULTIPLE REFLECTIONS

#### Theoretical principle

In this approach, backscattering due to sound reflection on vertical obstacles is considered by using a complementary Kirchhoff approximation [3] (called GFPE-Kirchhoff). "Complementary" means that the principle is the same as in the case of diffraction by a straight barrier (a series of receivers at the barrier calculation step have their pressure set to zero) but with the introduction of an image-source and with the complementary series of receivers.

This image-source S' is constructed relatively to barrier vertical plane. The sound pressure at any calculation point above the obstacle is set to zero (Fig. 1). If the barrier is not rigid, the calculated fields on its surface are multiplied by the plane wave reflection coefficient determined from the material impedance values and then propagated to the receiver [4, 5].

Let a wind blow from source S to receiver R. To insert the atmospheric refraction in the GFPE-Kirchhoff method, an upwind sound speed profile is also considered: this profile which is the symmetrical downwind one (relatively to neutral sound speed  $c_0$ ) is applied for the propagation from the image source S' to the obstacle. Then the "initial" downwind sound speed profile is used for the propagation from the obstacle to the receiver (Fig. 1).

The total pressure at the receiver is the sum of the "direct field above ground" calculation (a) and the one obtained in case (b).

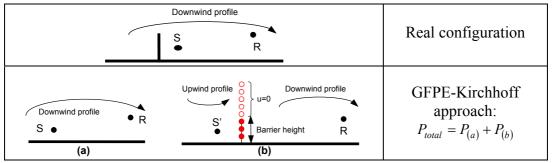


Fig. 1 - GFPE-Kirchhoff method applied to a barrier located behind the source in the case of a downwind profile (wind blowing).  $\circ u = 0$ ,  $\bullet u = u(r,z)$ 

The GFPE-Kirchhoff method can be easily extended to the case of multiple reflections due for instance to the presence of 2 parallel barriers [4,5]

## **Numerical validation**

A realistic road traffic noise configuration with a rigid barrier and an impedance jump is studied with meteorological effects as shown in Fig. 2 ( $\sigma = 180 \text{ cgs}$  Rayls with an infinite thickness, Delany and Bazley's formulation [6] for ground). A strong sound speed gradient described by the law  $c(z) = c_0 (1 + 4.9 \times 10^{-3} z)$  corresponding to a wind blowing from source to receiver is chosen to point out the influence of refraction.

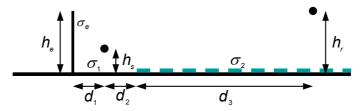


Fig 2 – Geometry of the barrier case with an impedance jump.  $h_S = 0.5 \text{ m}$ ,  $h_R = 4 \text{ m}$ ,  $h_e = 3 \text{ m}$   $d_1 = 7 \text{ m}$ ,  $d_2 = 7 \text{ m}$ ,  $d_3 = 90 \text{ m}$ ,  $\sigma_1 = \sigma_e = \infty$ ,  $\sigma_2 = 180 \text{ kPa s m}^{-2}$ ,  $c(z) = c_0 \left(1 + 4.9 \times 10^{-3} z\right)$ 

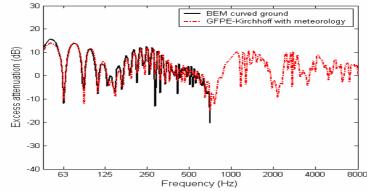


Fig. 3 - Excess attenuation vs frequency. Comparison between BEM and GFPE-Kirchhoff calculations results in inhomogeneous atmosphere for the case described in Fig. 2

Figure 3 shows the results obtained with the GFPE-Kirchhoff approach in the case of inhomogeneous atmosphere compared to the Boundary Element Method in a homogeneous atmosphere with an equivalent curved ground. The agreement between the two methods is very good.

## COMPLEX GEOMETRIES IN THE SOURCE VICINITY

We present here a BEM-GFPE hybrid method [4, 5] where the PE model is coupled with a Boundary Integral approach [7, 8]. On one hand, BEM is a powerful method able to calculate sound pressure at any point of the space with complex topography but it does not deal with meteorological effects. On this other hand, GFPE is efficient for long range sound propagation in inhomogeneous medium but no more when geometry is complex. The aim here is to take advantage of both methods.

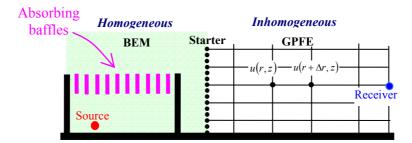


Fig.4 - BEM-GFPE hybrid method applied to the case of a vented cover between barriers

Calculations are divided in 3 steps (Fig. 4). First, the sound pressure in the vicinity of source and obstacle (barrier, complex protection, berm) is computed with the 2D-BEM neglecting meteorological effects. Then, the calculated sound field over a vertical line is adapted from 2D to 3D, and introduced as the starter for the PE. Finally, GFPE is used for the long-range propagation up to the receiver above a flat ground with meteorological effects and possible impedance jumps. This method can be extended to any complex configuration.

#### **Numerical validation**

A T-shaped barrier and a trench configuration have already been numerically validated elsewhere [4, 5]. Here the complex case of a vented road cover between two parallel barriers is investigated. The geometry is presented in Fig. 5, the surface impedances of the vertical baffles being calculated with the Delany and Bazley's formulation [6] with a thickness of 0.1 m and for  $\sigma_b = 30 \text{ kPa s m}^2$ .

Calculations are carried out without and with meteorological effects represented by a logarithmic sound speed profiles:  $c(z) = c_0 + \ln(1 + z/z_0)$  with  $c_0 = 340 \, m.s^{-1}$  and  $z_0 = 0.1 \, \text{m}$ . Results obtained from BEM-GFPE and BEM simulations are very close to each other in homogeneous medium. In the case of inhomogeneous atmosphere, the effects of meteorological conditions are clearly pointed out (Fig. 6).

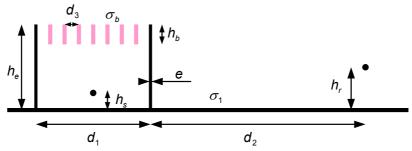


Fig. 5 - Geometry of the vented cover case.  $h_S = 0.5$  m,  $h_R = 1.5$  m,  $h_e = 8$  m,  $h_b = 1.5$  m,  $d_1 = 18$  m,  $d_2 = 15$  to 115 m,  $d_3 = 1$  m, e = 0.1 m,  $\sigma_1 = \infty$ ,  $\sigma_b = 30$  kPa s m<sup>-2</sup>

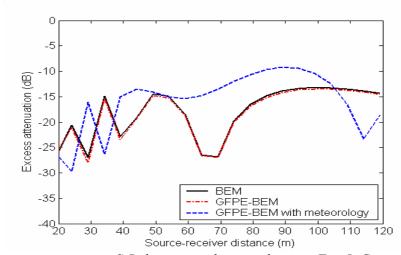


Fig. 6 - Excess attenuation vs S-R distance in the case shown in Fig. 5. Comparisons between BEM and BEM-GFPE calculations at 1000 Hz in homogeneous and inhomogeneous conditions with  $c(z) = 340 + \ln(1 + z/0.1)$ 

# **Experimental validation**

A measurement campaign above a 1/20 T-barrier scale model on a concave surface (ray of curvature: 10.2 m) has been undertaken to validate the theoretical results on flat ground in inhomogeneous atmosphere with  $c(z) = 340(1 + 4.9 \times 10^{-3} z)$  (Fig. 7).

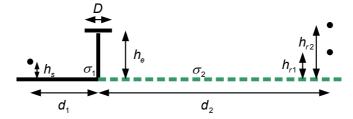


Fig. 7 - Geometry of T-shape barrier case.  $h_S = 0.5$  m,  $h_{R1} = 1.5$  m,  $h_{R2} = 4$  m,  $h_e = 3$  m,  $d_1 = 7$  m,  $d_2 = 75$  m, D = 1.2 m,  $\sigma_1 = \infty$ ,  $\sigma_2 = 180$  kPa s  $m^{-2}$ ,  $c(z) = 340 \left(1 + 4.9 \times 10^{-3} z\right)$ 

Meteorological effects are introduced using the analogy between sound propagation above a flat surface along curved ray paths and sound propagation above a curved surface along straight ray paths [9]. Scale model measurements are performed between 1000 and 20000 Hz which corresponds to a frequency range of 50–1000 Hz at scale 1/1. Grassland-like absorption is achieved by using a layer of felt ( $\sigma = 3600 \text{ kPa s m}^{-2}$ ) corresponding to  $\sigma = 180 \text{ kPa s m}^{-2}$  at 1/1 scale. The measurement method is the sine swept technique [10].

Comparisons between measurements (curved ground, homogeneous atmosphere) and BEM-GFPE (ATMOS) calculations (flat ground, downwind conditions, as in Fig. 7) are presented in Fig. 8. The agreement is good.

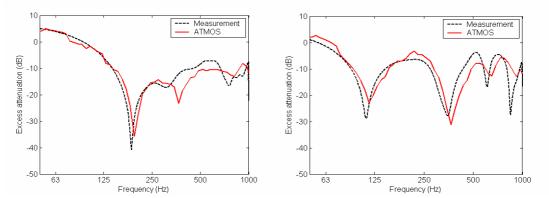


Fig. 8 - Excess attenuation vs frequency in the case shown in Fig. 7. Comparisons between scale model measurements and BEM-GFPE (ATMOS) calculations in inhomogeneous conditions. Results for receivers R1 (left) and R2 (right)

# **UNEVEN GROUND**

Several methods already exist to take uneven ground into account in the PE [11-13]; however none is dedicated to the study an embankment with the GFPE approach. In the present work the terrain is approximated by a succession of flat domains (Fig. 9). Above each of them, the sound field is propagated from an initial vertical sound field starter perpendicular to the slope. To switch from one slope to the other, the sound field is calculated at the tilted boundary between two successive zones.

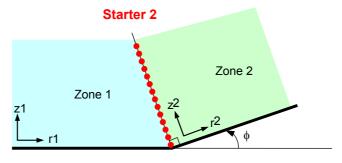


Fig. 9 - Reference rotation approach applied to an embankment (positive slope)

This approach can be extended to more complex topographies with several successive zones [14]. The same principle is applied for each new rotation of calculation domain.

#### **Numerical validation**

A validation in the case of a receiver on an embankment has already been presented elsewhere [4, 5]. Here the case of a source on the embankment is investigated (Fig. 10) which involves 3 successive zones.

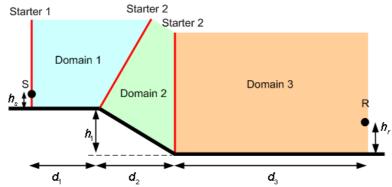


Fig. 10 - Geometry of the embankment case.  $h_S = 0.5$  m,  $h_R = 1.5$  m,  $h_I = 4$  m,  $d_I = 7$  m,  $d_2 = 6$  m,  $d_3 = 58.4$  m; ground is rigid

First, calculations in homogeneous medium have been carried out for the case shown in Fig. 11. Results obtained with modified reference rotation GFPE (called also GFPE-Topo) have been compared to the BEM ones in terms of excess attenuation (Fig. 11, left). The agreement between both methods is very good.

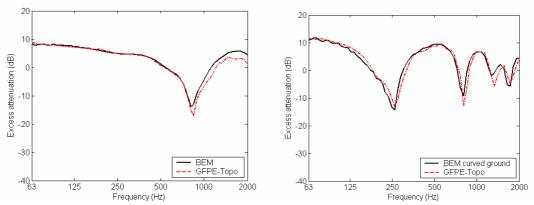


Fig. 11 - Excess attenuation vs frequency. Comparison between BEM and GFPE-Topo (ATMOS) calculations in homogeneous conditions (left) and inhomogeneous with  $c(z) = 340(1 + 4.9 \times 10^{-3} z)$  (right)

In a second step calculations have been completed in downward conditions (Fig. 11, right) in order to point out the effects of such meteorology on sound propagation. Agreement between new method calculations (GFPE-Topo) and reference ones (BEM with curved ground) is very satisfactory again.

#### **CONCLUDING REMARKS**

Results show that the modified GFPE approaches (ATMOS) are efficient in numerous complex road traffic noise configurations not solvable with classical PE or BEM. It is adapted to take multi-reflections as well as uneven ground and complex noise protections into account with coupled meteorological effects. Comparisons with reference numerical simulations and scale model measurements have shown a good agreement. Work is still in progress to investigate complete 3D propagation problems and to better understand the terrain outcome on the aerodynamic effects and thus use more realistic range dependant wind speed profiles for long range sound propagation.

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